ON THE QUANTUM EVOLUTION OF CHAOTIC SYSTEMS AFFECTED BY REPEATED FREQUENT MEASUREMENT

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Abstract. We investigate the effect of repeated measurement for quantum dynamics of the suppressed systems which classical counterparts exhibit chaos. The essential feature of such systems is the quantum localization phenomena strongly limiting motion in the energy space. Repeated frequent measurement of suppressed systems results to the delocalization. Time evolution of the observed chaotic systems becomes close to the classical frequently broken diffusion-like process described by rate equations for the probabilities rather than for amplitudes.

1. Introduction

Evolution of quantum systems, when they are not being observed, are described by the Schrödinger equation. Measurements change abruptly the state of the system and project it to the eigenstate of the measured observation. Repeated frequent measurement can inhibit or even prevent the quantum dynamics. This is the essence of the quantum Zeno effect or the quantum watched pot [1-3].

The quantum Zeno effect means that time evolution of the system from an eigenstate of the measured observable into a superposition of eigenstates is inhibited by the measurement. It has been shown in a recent experiment [3], that a variation of the quantum Zeno effect in a three level system, originally proposed by Cook [2], can be realized. Fierichs and Shenzle [4] have obtained the outcome of the experiment [3] on the basis of the standard three-level Bloch equations without the *ad hoc* collapse of the wave function.

Aharonov and Vardi [5] showed that frequent measurement can not only stop the dynamics of a system, but also may induce a time evolution of a system: a dense sequence of measurements along a presumed path induces a dynamics of the system close to this arbitrary chosen trajectory (see, also [6]).

One of the specific features of the systems mentioned above is that they consist only of the few (two or three) quantum states and are purely quantum. The purpose of this report is to consider the influence of the repeated frequent measurement on the evolution of the multilevel quasiclassical systems which classical counterparts exhibit chaos. It has been established that the chaotic dynamics of such, e. g. strongly driven by a periodic external field non-linear systems, is suppressed of the quantum interference effect and results to the quantum localization of the classical dynamics in the energy space of system [7,8]. Thus, the quantum localization phenomena strongly limits the quantum motion.

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Here we investigate the effect of the repeated measurement for quantum dynamics of suppressed systems. Our investigation is based on the mapping equations of motion for systems with one degree of freedom perturbed by a periodic force.

2. Quantum maps

From the standpoint of an understanding of manifestation of the classical chaos in the quantum description of systems, the region of large quantum numbers is of greatest interest. Here we can use the quasiclassical approximation and convenient variables are the angle ϑ and the action I. Transition from the classical to the quantum (quasiclassical) description can be undertaken replacing I by the operator $\hat{I} = -i\frac{\partial}{\partial\vartheta}$ [9,10]. (We use the system of units with $\hbar = m = e = 1$). One of the simplest systems in which dynamic chaos and quantum localization of states can be obtained is a system with one degree of freedom described by Hamiltonian $H_0(I)$ and perturbed by periodic kicks:

$$H(I, \vartheta, t) = H_0(I) + K \cos \vartheta \sum_{j} \delta(t - jT). \tag{1}$$

Here T and K are the period and parameter of the perturbation. The intrinsic frequency of the unperturbed system is $\Omega = dH_0/dI$. In particular for a linear oscillator we have $H_0 = \Omega I$. For $H_0 = I^2/2$ we have the widely investigated rotator [7,8].

Integration of the Hamilton equations of motion over one perturbation period leads to the classical map for the action and angle [9-11]:

$$\begin{cases}
I_{j+1} = I_j + K \sin \vartheta_j \\
\vartheta_{j+1} = \vartheta_j + \Omega(I_{j+1})T.
\end{cases}$$
(2)

For derivation of quantum equations of motion we expand the state function $\psi(\vartheta,t)$ of the system in quasiclassical eigenfunctions, $\varphi_n(\vartheta) = e^{in\vartheta}/\sqrt{2\pi}$, of the Hamiltonian H_0 ,

$$\psi(\vartheta, t) = (2\pi)^{-1/2} \sum_{n} a_n(t) e^{in\vartheta}.$$
 (3)

Integrating the Schrödinger equation over a period T, we obtain the following maps for the amplitudes [10]

$$a_n(t_{j+1}) = e^{-iH_0(n)T} \sum_m a_m(t_j) J_{m-n}(K), \quad t_j = jT$$
 (4)

where $J_m(K)$ is the Bessel function.

The form (4) of the map for the quantum dynamics is rather common: similar maps may be derived, e. g., for the atom in a microwave field [10,12].

3. Dynamics

Classical dynamics of the system described by map (2) in the case of global distinct stochasticity is diffusion-like with the diffusion coefficient in the I space resulting from (2),

$$B(I) = K^2/4T. (5)$$

From equations (4) we have the transitions probabilities $P_{n,m}$ between n and m states during the period T:

$$P_{n,m} = J_{m-n}^2(K). (6)$$

Using the expression $\sum_n n^2 J_n^2(K) = K^2/2$ and approximation of uncorrelated transitions we may formally evaluate the local quantum diffusion coefficient in the n space (see also [13,14]):

$$B(n) = \frac{1}{2T} \sum_{m} (m-n)^2 J_{m-n}^2(K) = \frac{K^2}{4T}.$$
 (7)

Therefore, the local quantum diffusion coefficient coincides with the classical one (5).

However, it turns out that such a quantum diffusion takes place only for some finite time t^* after which an essential decrease of the diffusion rate is observed. Such a behaviour of the quantum systems in the region of strong classical chaos was called "the quantum suppression of classical chaos". This phenomenon turns out to be typical for models (1) with the nonlinear Hamiltonians $H_0(I)$ and other quantum systems. Thus, the diffusion coefficient (7) derived in the approximation of uncorrelated transitions (6) does not describe the true quantum diffusion in the energy space. The quantum interference effect is essential and results to the quantitatively different dynamics. This is due to the pseudorandom nature of the phases $H_0(n)T$ in equation (4) as functions of n (but not j). Replacing $\exp[-iH_0(n)T]$ in (4) by $\exp[-i2\pi g(n)]$, where g(n) is a sequence of random numbers that are uniformly distributed in the interval [0,1] we observe the quantum localization as well [10]. However, the essential point is the independence of the phases $H_0(n)T$ on the step of iteration or time t_j . For the random phases as functions of iteration steps we observe the unlimited motion in the n space.

4. Influence of measurements

Each measurement of the energy projects the system into one of the energy eigenstates with definite n. Therefore, if we make a measurement before the next kick we will find the system in the states φ_m with the appropriate probabilities $P_m = |a_m|^2$. After the measurement the phase of the amplitude $a_m(t_j)$ is random. Thus, if there are measurements of the system's state before or after each kick, we have from equations (4) the system of rate equations for probabilities

$$P_n(t_{j+1}) = \sum_m J_{m-n}^2(K) P_m(t_j).$$
 (8)

The rate equations (8) result to the unlimited diffusion-like motion in the n space with the diffusion coefficient (7). Therefore, the frequent measurement of the quantum system described by map (4), which exhibits the quantum suppression of classical chaos, results to the diffusion-like motion, similar to the classical dynamics.

The analogous conclusion may be drawn and for other systems, as well.

5. Conclusions

Repeated frequent measurement of the simple two- or three-state system inhibit or even prevent the quantum dynamics of the system. The similar measurement of the multilevel quasiclassical system with quantum suppression of classical chaos results to the dynamics described by rate equations for probabilities rather than for amplitudes and causes the delocalization of the states superposition. The quantum dynamics of such chaotic systems affected by repeated frequent measurement resembles the classical diffusion-like motion.

It should be noted that the same effect may be derived without the *ad hoc* collapse hypothesis but from the quantum theory of irreversible processes. Even the simplest detector follows irreversible dynamics due to the coupling to the multitude of vacuum modes. The quantum system attains irreversibility due to the interaction with the detector and the phases of the amplitudes or the off-diagonal matrix elements of the density matrix decay steadily (see for analogy [4]). Measurement of such a type may also result to the disappearance of the effect of the quantum suppression. Therefore, the quantum evolution of chaotic system interacting with the environment or detector is more classical-like than the evolution of the idealized isolated system.

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